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trived to eliminate them, but by a process so long and complicated, that it is extremely difficult to follow him through the numerous transformations he has employed, and to see the reasons of them. This part of the theory is greatly simplified in the memoir, by developing only relatively to the quantities which it is our object to get rid of: we can develope with reference to the other quantities equally well, and even with advantage afterwards. The developments in question are effected with regard to the powers of the disturbing force, and not in series of sines and cosines.

It is easier to find the radius vector and longitude on the plane of the orbit than by referring the motion to a fixed plane; but it is a matter of more difficulty to find the latitude in this case. In this part of the theory the author has introduced changes which may be employed very advantageously when the inclination is small. The inclination (i) and the longitudes (ϑ and θ) of the node on the two planes produce many terms in the disturbance function. In place of these quantities, the latitude and reduction (the two sought quantities) are, in the memoir, introduced into this function, and into its partial differential coefficients. Afterwards sin i and sin $(v-\theta)$ are obtained, instead of the three quantities, ι, θ, ϑ, or p, q, and $\vartheta-\theta$, which M. Hansen finds. Thus, the latitude is found as readily as when the motion is referred, in the first instance, to a fixed plane.

The memoir concludes with a transformation of the differential equations from a fixed to a sliding plane, preserving a constant inclination (that of the orbit) to the fixed plane, and sliding upon it with a uniform motion, equal to the mean motion of the node. No details are added here; the transformation is merely noticed as appearing to merit further consideration.

Rev. W. Roberts read a paper, from which the following is an extract.

"In a note communicated some time since to the Academy, I extended to the system of any hyperbola and its conjugate a property of the equilateral hyperbola, and the lemniscate derived from it, given by Mr. Talbot, in the fourteenth volume of M. Gergonne's *Annales de Mathématiques.* The result at which I then arrived I have since found to be a very particular case of a curious and general theorem, which may be enunciated as follows:

"*Being given a hyperbola, the equation of which is*

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1,$$

where a is supposed greater than b, let the curve be described, which is the locus of the feet of perpendiculars dropped from the centre upon its tangents: from this new curve let another be derived, and so on, by repeating continually the above-mentioned construction, and let S_n *denote the perimeter of the* n^{th} *curve of the series. Also, let* Σ_n *be the perimeter of the* n^{th} *curve, obtained by a similar mode of generation from the conjugate hyperbola,*

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1.$$

Then, any combination of these perimeters, such as

$$S_{2i}\,S_{2i'+1}+\Sigma_{2i}\,\Sigma_{2i'+1},$$

will be expressible by elliptic functions of the first two kinds in the following manner:

$$S_{2i}\,S_{2i'+1}+\Sigma_{2i}\,\Sigma_{2i'+1}=\pi\{\alpha+\beta\mathrm{F}(k,\phi)+\gamma\,\mathrm{E}(k,\phi)\},$$

where

$$k^2=\frac{a^2}{a^2+b^2},\quad \cos\phi=\frac{b^2}{a^2},$$

and where α, β, γ, *are algebraic functions of a and b.*

"*The foregoing equation holds for the case of* $i=0$, *by supposing that* S_0 *(or* Σ_0*) expresses four times the difference between the infinite hyperbolic arc and its asymptote.*

"It is evident that we may give a purely geometrical enunciation to the above theorem. For let s denote the arc of the hyperbola,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1,$$

measured from the vertex to the point of which the ordinate (y) is $\sqrt{(a^2 - b^2)}$; and let s' be the arc of the ellipse,

$$\frac{x^2}{a^2 + b^2} + \frac{y^2}{b^2} = 1,$$

counted from the extremity of the lesser axis to the point of which the ordinate is $\frac{b^3}{a^2}$; and we will have

$$S_{2i}\, S_{2i'+1} + \Sigma_{2i}\, \Sigma_{2i'+1} = \pi\{\alpha + \beta s + \gamma s'\},$$

α, β, γ, being, as before, algebraic functions of a and b.

" By making $i = i' = 0$, we light on the theorem which I formerly communicated to the Academy. In this case γ, in the last equation, vanishes.

" The above theorem, which I think there would be some difficulty in establishing directly, I have demonstrated very simply, by employing the method of elliptic co-ordinates, with the vast power of which, as an analytical instrument, all readers of the mathematical publications of the present day must be familiar. I subjoin a sketch of my proof, which, I trust, may not be altogether uninteresting.

" In the first place, I calculate the values of S_{2i}, $S_{2i'+1}$, Σ_{2i}, $\Sigma_{2i'+1}$, by means of a general formula which I obtained some years since, and which has appeared in the Proceedings of the Academy. Then, after a few easy transformations, I am enabled to write the sum of the two products, $S_{2i}\, S_{2i'+1}$ and $\Sigma_{2i}\, \Sigma_{2i'+1}$, as a double definite integral, which, although consisting of a variety of terms, is found ultimately to depend on two only, which are distinct and independent. Of these, one expresses the superficial area of a certain ellipsoid, and the other, the sum of all the superficial elements of the same ellipsoid, divided by the squares of the areas of the sections of the surface made by the diametral planes parallel to these elements. The reduction of these integrals to the normal form of elliptic functions may be effected in a variety of ways; the simplest, probably, is that given by M. Jacobi, for the rationalization of such integrals, in the tenth volume of Crelle's Journal.

" The combination, $S_{2i} S_{2i'+1} + \Sigma_{2i} \Sigma_{2i'+1}$, of the perimeters of the curves derived from conjugate hyperbolæ, which, as we have seen, admits of being put under such a remarkable geometrical form, has also a very curious signification in mathematical physics. This remark I owe to my distinguished friend, M. Liouville, who mentioned it to me in conversation a short time since."

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DECEMBER 11TH, 1848.

REV. HUMPHREY LLOYD, D. D., PRESIDENT,
in the Chair.

A COLLECTION of stone, bronze, and iron antiquities, with some casts of specimens in the Museum at Copenhagen, were presented by the Royal Society of Northern Antiquities.

The Rev. Dr. Todd (Secretary) directed the attention of the meeting to a highly interesting group of antique relics, which had been presented to the Academy by the Royal Society of Northern Antiquities at Copenhagen. He observed that those specimens were, some of them, analogous to antique remains of a similar character which had been found in Ireland, several of which were in the possession of the Academy. The existence of such an analogy between the weapons and instruments used in ancient times, by the inhabitants of this and more northern countries, was known to the Academy, and a small collection of antiques of this nature, found in Ireland, had been selected and transmitted by the Academy to the Northern Society, along with a large collection of drawings